RESEARCH STATEMENT

JACOB LAUBACHER

My research lies in algebra. In particular, I spent my graduate work enveloped in homological algebra and category theory, whereas more recently I have begun work in group theory. I also prefer to make time to work with undergraduates on suitable topics and projects.

Homology first got its start through applications in topology. Besides distinguishing two topological spaces, homological algebra has had many other applications, spanning to areas of Lie algebras, differential geometry, and noncommutative algebra, among others. Roughly speaking, homological algebra is a way to classify obstructions that need to be overcome in order to continue a construction. Much of modern homological algebra can be attributed to Cartan, Eilenberg, and Mac Lane in the 1940's, with the foundation being solidified with category theory shortly thereafter. The 1960's brought derived functors and derived categories, introduced by Grothendieck and Verdier, which had far-reaching consequences and expanded the subject. One can see [14] for more reading.

On the other hand, group theory (and more specifically character theory) was expanded and developed by Frobenius in the 1890's. This area of mathematics was stabilized by Brauer and Isaacs, and much of what I have worked on is an immediate consequence of Lewis (see [8]).

Homological Algebra

In [13], myself, my advisor, and another expert in the field investigated the simplicial structure of the chain complex associated to the (secondary) Hochschild (co)homology. Besides developing the necessary theory, the main ingredient was a simplicial object which behaved similar to the well-known bar resolution associated to an algebra. In [5] we were able to generate a bar-like resolution for the higher order Hochschild (co)homology of a commutative algebra A with values in the A-symmetric A-bimodule M over the d-sphere (for any $d \geq 1$). We even explore the noncommutative setting in [6] and deformation theories in [4].

Many of the details for [13] and the case $d = 2$ from [5] are in my dissertation [9]. In [10] and [11] we focus on specific properties of the secondary Hochschild homology: Morita equivalence, an exact sequence, a correspondence with Kähler differentials, the kernel of a multiplication map, functoriality.

Group Theory

Recently, I decided to pursue furthering my knowledge in group theory. We worked in character theory and representation theory, and began with the goal to finish the classification of character degree graphs of solvable groups with six vertices (see [2]). Along the way we were able to classify a family of graphs, which was as exciting as it was unexpected. In [1] we determine completely whether or not any graph in a certain family occurs as the prime character degree graph of a solvable group. We expand on these constructions of families in [12] and its sequel [7], and have more avenues for the future. Finally, we have continued in this area of character theory, and now we have explored certain conditions that a prime character degree graph can possess in order to guarantee there is no associated normal nonabelian Sylow p-subgroup. This

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is detailed in [3]. Tangentially, I have also been fortunate enough to work with undergraduates on projects. I anticipate continuing my involvement with undergraduates in the future, and even have several projects lined up with curious undergraduates.

Future Work

There are plenty of questions that I have left unanswered. Most of these occupy me still, though one can argue whether or not I am making progress. More importantly, I have found that working with others, regardless of where they are, seems (to me) more preferable. I have appreciated my collaborations with coauthors. Besides my ongoing work, I have enjoyed my time working on projects with undergraduates. I have plenty of ideas worth exploring, be it involving the Rubik's cube, games like chess, graph theory, or a topic the student brings to me. Even if what we do with undergraduates is not new and original, it will be a growth in their mathematical abilities, and it is one that I cherish and look forward to continuing.

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