The Kool-Aid Man

Jake Laubacher

St. Norbert College

Math Colloquium Series – November 11, 2020

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Graduation

The Parents

In the Kitchen

A Typical Monday

(Morning)
Mom: What do you have planned today?
Jake: I'm going to sit here and do some math.
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Mom: You should get a girlfriend.
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Mom: You should get a girlfriend.
Jake: Math is my girlfriend!
Mom: MATH CAN'T GIVE ME GRANDKIDS!!

Character Degree Graphs

- Vertices: all primes which divide elements of our set $\mathrm{cd}(G)$
- Edges: there is an edge between distinct vertices p and q if there exists some $d \in \mathrm{cd}(G)$ such that $pq \mid d$

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Some Properties

The graph is "simple" (no loops, no double edges, no direction).
The graph doesn't have to be connected.

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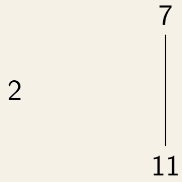

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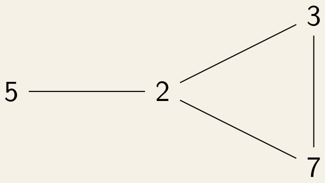

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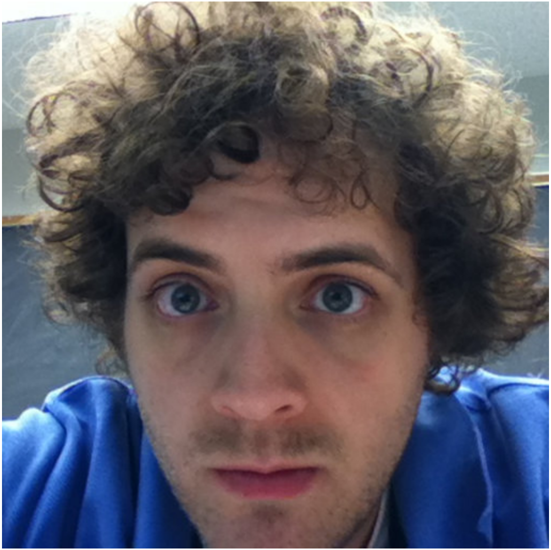
"We need a life jacket!"

Mark



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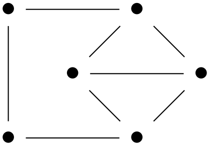
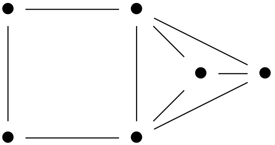
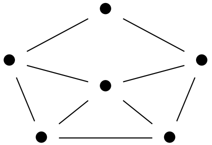


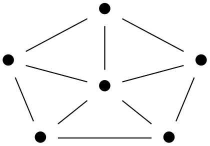
(*i*)



(*ii*)



(*iii*)



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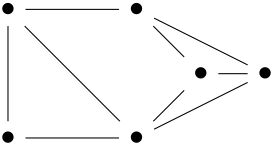

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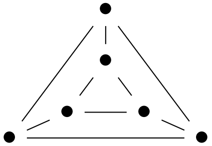

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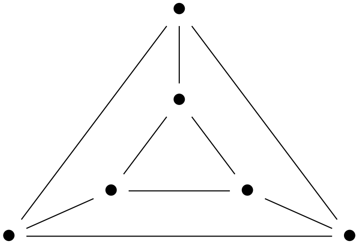

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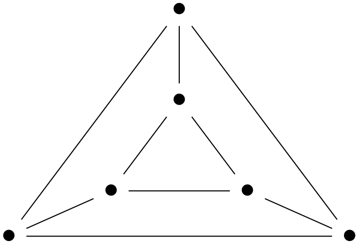

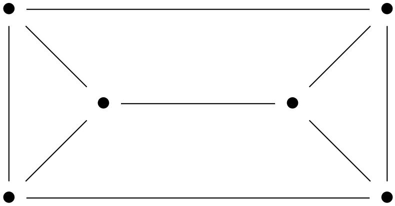

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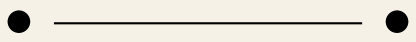

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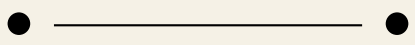

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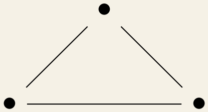

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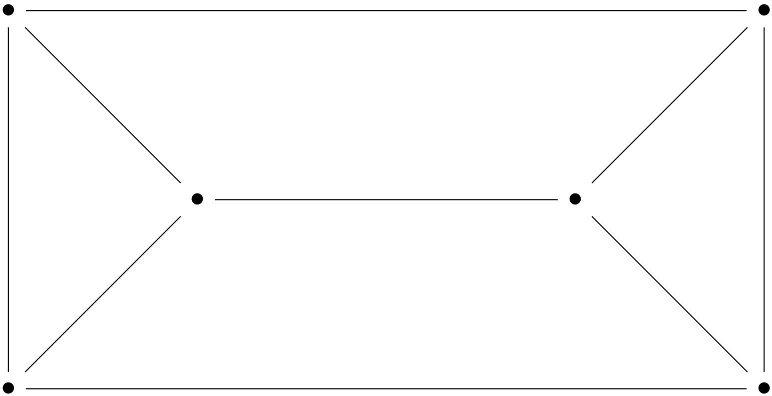

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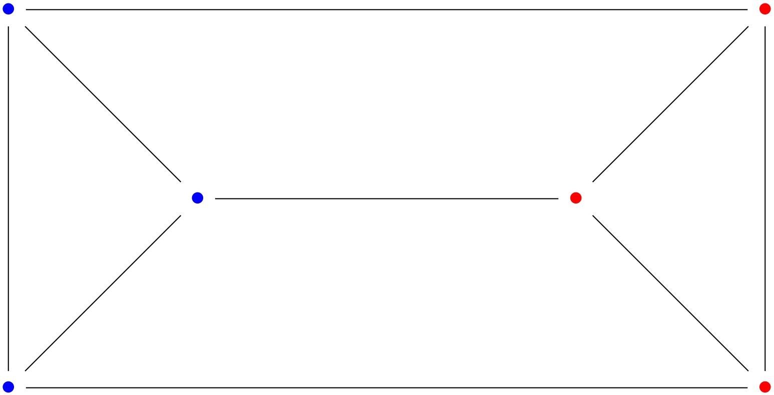

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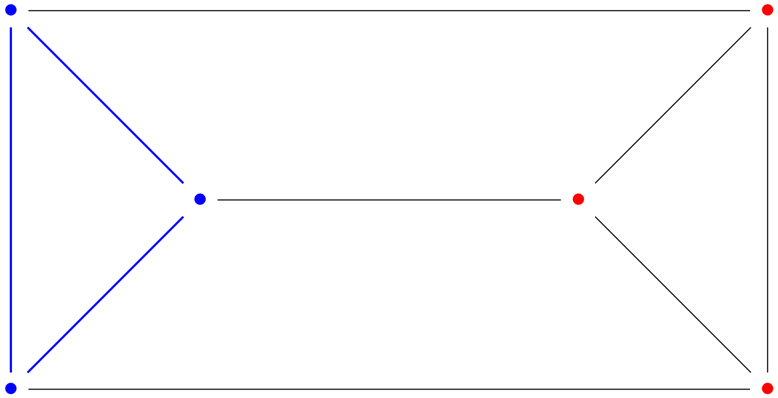

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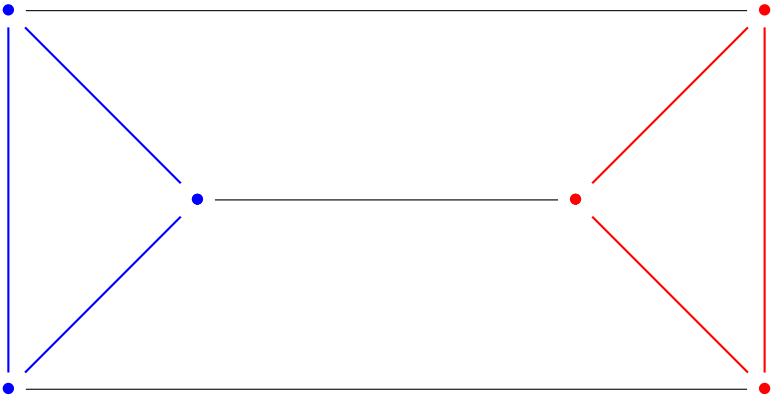

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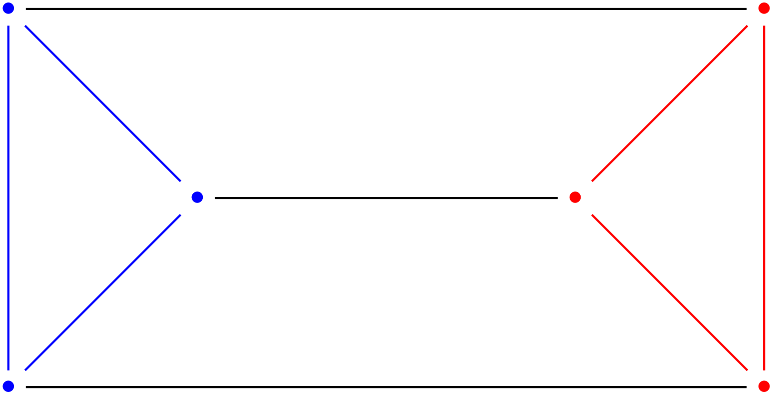
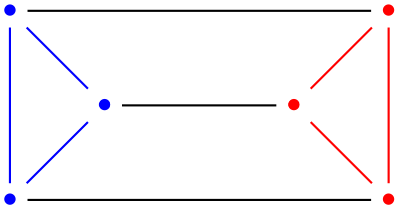


A Pattern



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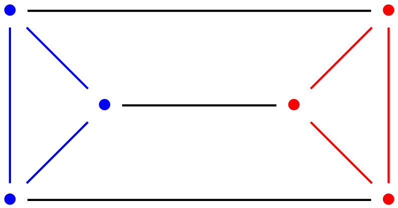

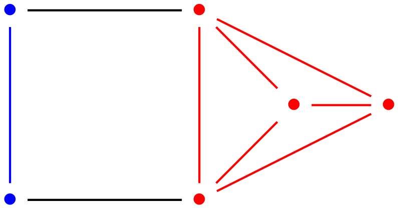

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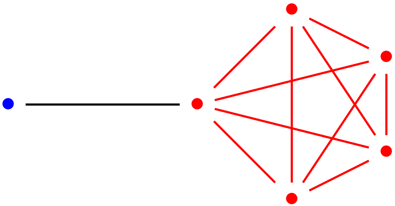

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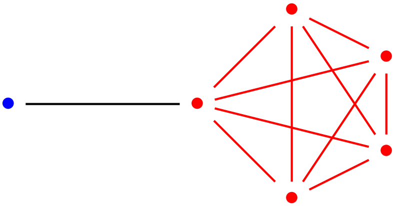

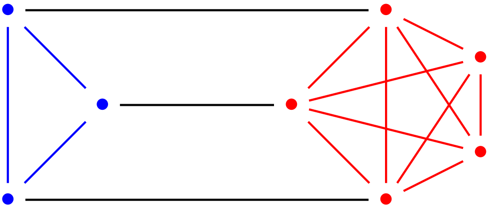

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Proof.

Induction. Twice. On both k and t. It was cool. □

Thanks!

Questions?